Bound for entropy & viscosity ratio of strange quark matter.

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Abstract

High energy density (ϵ) and temperature (T) links general relativity and hydrodynamics leading to a lower bound for the ratio of shear viscosity (η) and entropy density (s). We get the interesting result that the bound is saturated in the simple model for quark matter that we use for strange stars at the surface for $T \sim 80~MeV$. At this T we have the possibility of cosmic separation of phases. At the surface of the star where the pressure is zero - the density ϵ has a fixed value for all stars of various masses with correspondingly varying central energy density ϵ_c. Inside the star where this density is higher, the ratio of η/s is larger and are like the known results found for perturbative QCD. This serves as a check of our calculation. The deconfined quarks at the surface of the strange star at $T = 80~MeV$ seem to constitute the most perfect interacting fluid permitted by nature.

1   Introduction

Strange stars are made of deconfined $u,~d,~s$ matter. The pressure at the star surface is zero with a surface number density is around $4-5$ times the normal matter density. The central density is almost 3 times the surface density. We find that the ratio of the kinetic viscosity to entropy density of strange stars (SS) nearly saturates the lowest possible bound found by Kovtun, Son and Starinets [1] (KSS in short) at the surface at high T. This is as perfect as an interacting fluid can be. The relevant T where this happens is where the cosmic separation of phases takes place [2]. This is in the sense that it is the critical T above which no zero pressure point exist for the deconfined quarks. This implies that above this T there can be no self bound strange stars. Below this T, the two phases of hadronic stars and

quark stars can both exist [3] as the surface tension of the strange stars is high [4]. The temperature estimated by Witten [2] for this was $T = 100 \, MeV$ which is close to what we get.

Our calculation is surprising to a certain extent, we try to confirm it by moving from the surface to the inside of the strange star. KSS state that somewhat counterintuitively, a near ideal gas has a large viscosity. In agreement with this observation, deep inside the star the condition are more like perturbative (or weak coupling) QCD and we find that η/s is larger than at the surface and comparable to the results of Arnold, Moore and Yaffe [5]. This is a consequence of the crucial density dependence of the quark mass that we have assumed and can be interpreted as a support of our assumption. We must stress however that the value of strong coupling constant α_s relevant for the KSS bound is large ~ 0.6.

We talk of shear viscosity that is relevant for the problem and the bulk viscosity is negligible at least for weak coupling as shown by Arnold, Dogan and Moore [6]. For values of $\alpha_s \sim 0.3$ the bulk viscosity is thousand times smaller that the shear viscosity. Interestingly they note that at high density where the QCD coupling is small, there are long lived quasiparticles and a kinetic theory treatment should be valid which we find to be valid also at larger α_s.

Many of the relevant points discussed in the literature are summarized in a recent review by Blaizot [7]. The experimental data from heavy ion collisions (RHIC) do not provide any evidence for ideal gas behavior, rather the produced matter behaves as a fluid with low viscosity, the "perfect fluid".

New techniques have emerged that allows calculations to be done in some strongly coupled gauge theories that differs however in essential aspects from QCD. The answer to the question - is quark -gluon plasma weakly or strongly coupled - does not have a straight forward answer. Indeed in the quark gluon plasma coexist seemingly perturbative features, and non perturbative ones. This is the view which matches with our spirit.

The background for the viscosity bound conjecture of KSS [1] will be briefly touched upon for the sake of completeness :

It is popularly known that black holes are endowed with thermodynamics. In higher dimensional gravity theories there exist solutions called black branes and they are black holes with translation-ally invariant horizons. For these solutions thermodynamics can be extended to hydrodynamics - the theory that describes long-wavelength deviations from thermal equilibrium. Applying the holographic principle a black brane corresponds to a certain finite-temperature quantum field theory in fewer number of space time dimensions, and the hydrodynamic behaviour of black-brane horizon is identical with the hydrodynamic behaviour in a dual theory.

The arguments of KSS for generalization of the viscous bound $4 \pi \eta/s > 1$ - is more interesting since it only invokes general principles like the Heisenberg uncertainty relation for the typical mean free time of a quasi-particle and the entropy density s. From here to our model is just one short step of identifying the quasi-particles to be the dressed quarks of a mean field description for a large colour effective theory. Further light in this direction comes from the recent work of Fouxon, Betschart and Bekenstein (FBB in short) [8] as we shall discuss later in this paper. For a black hole calculation for the matter inside is of course impossible so FBB concentrate on the generalized second law of thermodynamics that they call GSL. Following them one can state that GSL claims that the sum of entropy of all the black holes and the total ordinary entropy in the black holes' exterior never decreases. Then they go on to consider a simple spherical accretion model and suggests that this Bondi flow satisfies GSL because the accretion velocity approaches the speed of light.

Our model is presented in the next section emphasizing the possible astrophysical observational checks that have already been discussed extensively in the literature. In section 3, we describe the calculation of the viscosity known to all. In section 4, the considerations enumerated by FBB are shown to be satisfied in our model and we present a summary and conclusion in the last section.

2   Strange stars at finite T

The density dependent quark mass is given in our model as :

$$M_i = m_i + M_Q \; sech\left(\frac{n_B}{Nn_0}\right), \quad i = u, d, s.$$ (1)

where $n_B = (n_u + n_d + n_s)/3$ is the baryon number density, $n_0 = 0.17 \; fm^{-3}$ is the normal nuclear matter density, and N is a parameter taken to be 3 in the set F of [3] which we have chosen here. The results for A-E are not too different as can be seen from Table 1 of [3]. For set F the maximum mass possible for SS is $1.436 M_\odot$ and the corresponding radius is $6.974 \, km$. At high n_B the quark mass M_i falls from a large value M_Q to its current one m_i which we take to be $m_u = 4 \; MeV$, $m_d = 7 \; MeV$, $m_s = 150 \; MeV$ [9]. M_Q is taken as $345 \; MeV$ in set F of [3]. Possible variations of chiral symmetry restoration at high density (CSR) can be incorporated in the model through N.

We use a modified Richardson potential with different scales for confinement ($\sim \; 350 \; MeV$) and asymptotic freedom ($100 \; MeV$) which has been tested by fitting the octet and decuplet masses and magnetic moments [10, 11] and the temperature dependence of the gluon mass is taken from Alexanian and Nair [12].

The finite T calculation involves a T-dependent gluon screening and thermal single particle Fermi functions with interactions that involve all pairs of quarks. Along with the painstaking constraints of β - equilibrium and charge neutrality in these calculations - it is found that zero pressure occurs at a density ~ 4 to 5 times the normal nuclear density n_0 till $T \;\; = \;\; 80 \; MeV$. This is a relativistic mean field calculation with a screened Richardson potential for two quarks, where only the Fock term contributes. The calculation is self consistent. Strange quark matter is self bound by strong interaction itself. The energy density and pressure of this matter lead to strange quark star through the TOV equation with mass and radius depending on the central density of the star.

The model has been applied to discussions on compactness of stars [9, 13, 14], quasi-periodic oscillations in X-ray power spectrum [15], the existence of minimum magnetic field for all observed pulsars [16], absorption and emission bands along with high redshift [17], superbursts [18] and high value of surface tension useful to stabilize the strange stars [4].

3   Calculations

We use the classical expression for evaluating the shear viscosity coefficient η as:

$$\eta = \frac{1}{3}mvn\lambda$$ (2)

where the mean free path λ is given in terms of the interaction diameter of quark d_q and the appropriate number density n

$$\lambda = \frac{1}{(4/3)nd_q^2}.$$ (3)

We need to specify the average momentum P which we take from the Fermi distribution

$$\langle P \rangle = mv = \frac{\int_0^\infty k^3 f(k, U_i) dk}{\int_0^\infty k^2 f(k, U_i) dk}, \quad i = u, d, s$$ (4)
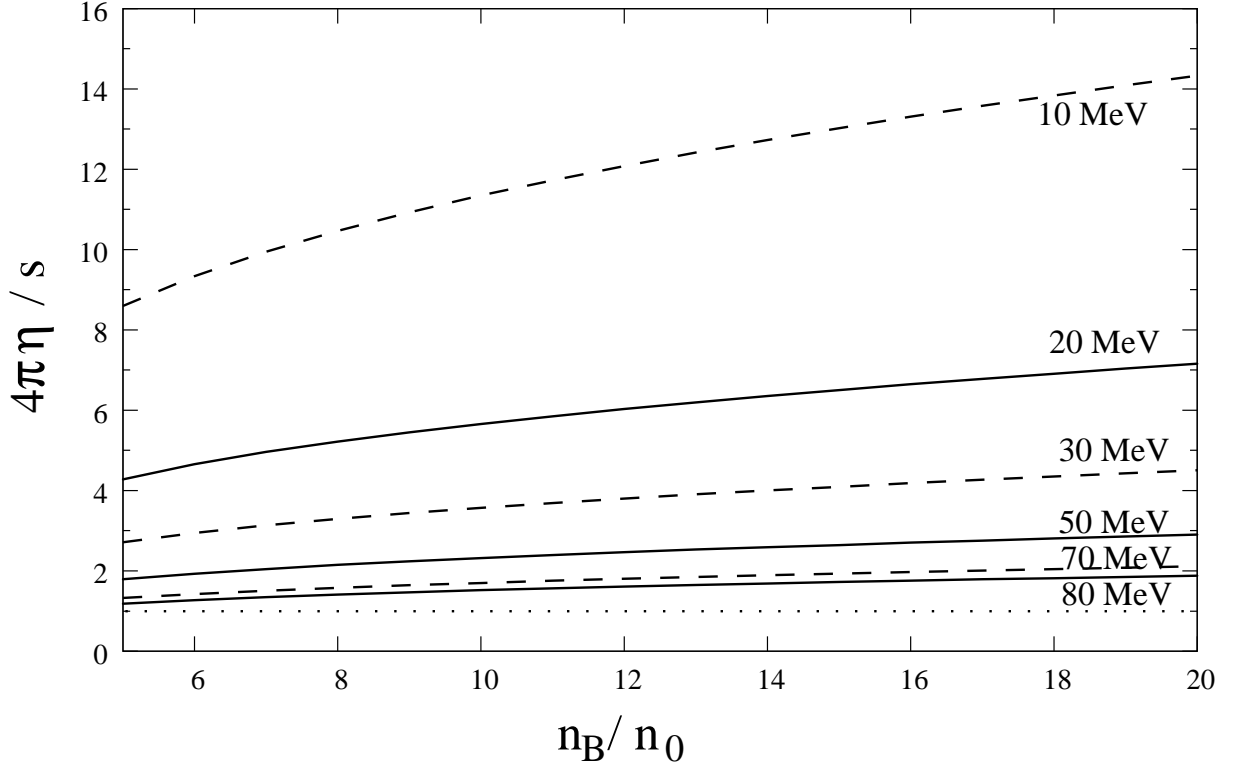
$$f(k, U_i) = \frac{1}{1 + exp[(U_i - \mu_i)/T]}.$$ (5)

Figure 1: 4π times shear viscosity η divided by the entropy density for various number density is plotted. According to the KSS bound [1] this should be one for what is called the most perfect fluid, perhaps encountered in RHIC [19]. We see that the bound is nearly saturated at $n_B/n_0 \sim 5$ which is the surface of the star at T = $80~MeV$.

Heiselberg and Pethick(1993) suggested that the quark scattering cross section πr^2 can be compared to proton-proton scattering using the quark counting rule $\sigma_{pp} = 3~\sigma_{qq} = 3\pi r_n^2$ [20] where r_n is the interaction radius. In matter this is calculated by assuming that the relevant particles (in this case the quarks) occupy an effective volume $\frac{4}{3}\pi~r_n^3$.

We calculate the diameter of the quarks d_q by assuming that they are packed tightly on the surface of the star. This is justified since the gravitation is strong and it will try to minimize the surface. The quarks, assumed to be spheres, have radius $r_q = d_q/2$ and their projected area on the surface of the star ($4\pi\ R^2$) will be $\pi\ r_n^2$ giving the number to be :

$$N_q = 4R^2/r_n^2. \tag{6}$$

The volume of the tightly packed layer is $V = 4\pi\ R^2 \times d_n$ and the number is $V \times n$ where the n is the self consistent number density corresponding to the definition of the zero pressure surface of our model. This number, equated to N_q given above, leads to :

$$d_n = \left[\frac{4}{\pi n}\right]^{1/3}, \tag{7}$$

The number density for the strange star in our model changes from the surface where it is between four and five times the normal nuclear matter density n_0 to about 15 times n_0 in the centre of the star for T = 0. For finite T the numbers increase somewhat due to the Fermi distribution.

We see in Fig.(1) that the $4\pi\eta/s \sim 1$ for the highest T where strange stars are self bound for the star surface which has the lowest value of the number density. At higher densities the ratio is much larger as is the case for perturbative QCD.

The variation of η/s with the coupling is counter-intuitive as emphasized by KSS. We wanted to check that the ratio in fact increases with decreasing coupling. To do this we needed the relevant α_s at each density.

We have extracted the strong coupling constant α_s from the density dependence of the mass given in eq.(1) as in [21, 22]. This is due to the simplified Schwinger-Dyson formalism of Bailin, Cleymans and Scadron using the Dolan-Jackiw Real Time propagator for the quark. We re-do the calculation here for the M_d and the $n = 3$ appropriate for our latest parameter set but essentially there is no fundamental change in α_s, the variation being from ~ 0.6 at low density to about 0.2 at the highest.

$$\alpha_s(r,n) = \frac{m_{dyn} - M_d(r,n)\pi}{2\ m_{dyn}\ \ln\left[\frac{\mu(r,n)+(\mu(r,n)^2-M_d(r,n)^2)^{.5}}{M_d(r,n)}\right]}. \tag{8}$$

The variation of $4\pi\eta/s$ with α_s has been shown in fig. 2. The interesting point here is that the value of $4\pi\eta/s$ is larger than one by factors ranging from 2 to 14 for various T at $\alpha_s \sim 0.2$ so that it is clear that transport of quarks is the main factor for the largeness of this factor and the smallness of the interaction does not matter.

In a recent paper Lacey has given a very lucid and colourful representation of viscosity bound for different fluids (see fig. 3 of [19]) which we summarize here. As the $(T - T_c)/T_c$ varies from -0.5 to 0, η/s in (a) meson gas goes from 1.2 to 0.4, (b) water goes from 3.8 to 2.2 (c) liquid nitrogen from 3.4 to 0.8 and (d) liquid helium from 3.4 to 0.8. The matter in the strange star seems to be the first so called perfect interacting liquid where bound reaches the fraction $\sim (4\pi)^{-1}$ and thus it may be the same fluid which Lacey marks as RHIC.

We would like to mention another recent paper dealing with boost-invariant viscous hydrodynamics [23] for although this deals with a theory which does not have a direct counterpart which works for QCD it may still be useful for studying features of the plasma that is strongly coupled and deconfined.
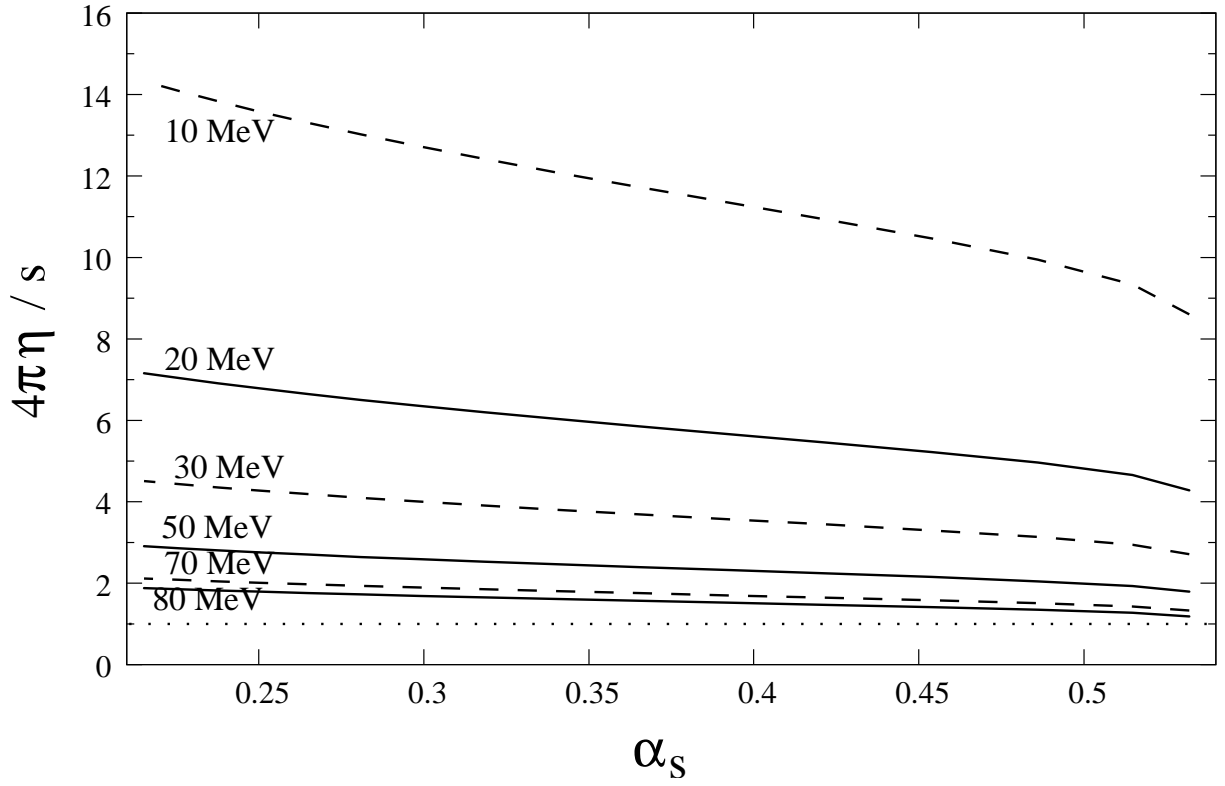
Figure 2: We also find that η is a decreasing function of coupling strength as discussed for example by Stephanov [25]. We should stress that the value of α_s relevant for this paper is large, about 0.65.

4 Bekenstein bound & its connection to that of KSS.

In a recent paper FBB [8] has suggested that the KSS bound is related to the Bekenstein bound [24]

$$S/E < 2\pi R \tag{9}$$

where R is the radius of the smallest sphere circumscribing a system whose entropy is S and energy is E and then they reduce it to what they call the UBE, the universal bound for entropy :

$$s/\epsilon < 2\pi\lambda \tag{10}$$

where s, ϵ are the entropy and energy densities respectively and λ is the mean free path. In Table(1) we present these quantities and it is clear that the inequality is satisfied T increases from 1 to 80 MeV, at the surface where quark number density varies from $2.04 \ fm^{-3}$ to $3.22 \ fm^{-3}$. At T = 90 MeV, which is the last entry, the eqn. (10) is just about violated and coincidentally a zero pressure point is no longer there in our equation of state.

The use of Bekenstein bound for RHIC is not new. The entropy bound has been invoked to set limits for T at which hadrons can survive as a confined system. For example, the pion may form at lower T than the ρ meson [26] and that the pion cannot exist at 90 MeV if its mass is 138 MeV and and its radius is 0.445 fm (see Table 2 of [26]). It is satisfying to see that the same temperature is invoked in strange quark matter with the updated Bekenstein bound Table (1).

Table 1: Comparing entropy-energy ratio with momentum at different temperature T. It may be noted that the Bekenstein bound as updated by FBB, namely $s/\epsilon \leq 2\pi\lambda$ is exactly satisfied as an equality between T = 80 and 90 (in MeV). Number density is n, d_q is the average interaction diameter of the quarks at the star surface, P is the average momentum and η is the kinetic viscosity.

T	$n(fm^{-3})$	$\epsilon(fm^{-4})$	$s(fm^{-3})$	$d_q(fm)$	$\eta(fm^{-3})$	$P(fm^{-1})$	$2\pi\lambda$	s/ϵ	$\eta/\epsilon\lambda$
1	2.04	3.202	.05615	.85459	.45972	4.2192	1.0068	.01754	.89599
10	2.0549	3.3528	.57913	.85251	.46544	4.2509	1.0043	.17273	.86845
20	2.1023	3.5843	1.1927	.84606	.48454	4.3585	.99674	.33276	.85215
30	2.1846	3.9036	1.8386	.83530	.51768	4.5390	.98407	.47101	.84672
40	2.3019	4.3221	2.5221	.82087	.56420	4.7774	.96706	.58351	.84812
50	2.451	4.8384	3.2535	.80387	.62202	5.0511	.94704	.67242	.85292
60	2.6255	5.438	4.0445	.78566	.68798	5.3365	.92558	.74375	.85882
70	2.8174	6.0988	4.9062	.76739	.75872	5.6147	.90406	.80446	.86459
80	3.0193	6.7975	5.8484	.74989	.83129	5.8743	.88344	.86037	.86976
90	3.2249	7.5131	6.8796	.73360	.90340	6.1096	.86425	.91568	.87417

One can proceed to find more interesting results. According eqn. (34) of FBB, $\eta \sim \epsilon\lambda a$ where a is the speed of sound. Thus the last column of Table (1) shows that the velocity of sound is close to the velocity of light. This is consistent with the findings of Sinha $et \ al$ [27] where a is calculated from

first principles by evaluating the incompressibility. As stated in our introduction luminal velocity of Bondi accretion flow $U_{ac} \sim 1$ was invoked by FBB and this is reminiscent of that.

At T = 80 MeV we have

$$s \ = \ 4\pi \, \eta \ = \ (4\pi/3) P \, n\lambda < 2\pi\epsilon\lambda \tag{11}$$

which yields the inequality for the average momentum $P \ < \ 1.5\epsilon/n$ where P is the average momentum and ϵ/n is the energy per particle. This can be directly compared with KSS who state that the energy of a quasiparticle and its mean free time τ_{mft} cannot be smaller that $\hbar$ and hence $\eta/s \ \geq \ \hbar/k_B$. Recalling that we work with units $k_B \ = \ \hbar \ = \ c \ = \ 1$ and that the quarks have velocities comparable with the velocity of light c one can see that both relations are consistent with the uncertainty relation. Thus it can be asserted that the generalized second law of thermodynamics and the uncertainty relation have some consistency checks if one uses the Bekenstein bound UBE and the KSS bound.

5   Discussion

We are grateful to the anonymous referee for raising an important question that what happens at a temperature higher than $\sim \ 100 \ MeV$ or a density much lower than 4-5 times the normal matter density ? The deconfined strange quark matter does not exist below the critical density of 4-5 times the normal matter density above a temperature of 80 MeV in our mean field model. In Witten's original scenario [2] for cosmic separation of phases - a QCD and a hadron phase started around 100 MeV. A different phase was implied above this temperature which was not specified. One could imagine this could be a pre-QCD phase or it could be hadrons overlapping with quarks percolating through. We propose that the hydrodynamics of such a phase will satisfy the KSS bound along the boundary of the density-temperature curve on which our point is a low temperature high density point whereas in RHIC a lower density and a higher temperature of 200 MeV may be obtained and will show the KSS bound. It is our conjecture that the KSS bound is always valid on this curve. To us this seems to be a likely scenario in view of the many model calculations done by many groups recently [28, 29].

6   Summary and Conclusions

η increases with increasing energy density $i.e.$ decreasing α_s for the matter that composes a self bound strange star. The transport here is radial hence η is the shear viscosity. At the surface of the star the pressure is zero and the number density is the same for stars of all masses. The quark matter at the surface saturates the bound given by [1] for $T \ = \ 80 \ MeV$ - the highest T where we get zero pressure.

Our model leads to such an interesting result, connecting zero pressure with the viscosity bound on the one hand and RHIC on the other hand. The updated Bekenstein bound is exactly satisfied as an equality at high density between T = 80 and 90 MeV where Witten's cosmic separation of phases is possible.

Acknowledgments

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